INFLUENCE OF THIRD STRESS DEVIATOR INVARIANT ON CREEP OF NONHARDENING MATERIALS

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Starting from the hypothesis of the existence of the creep strain rate potential function and following the ideas developed in [1, 2], we propose a particular form of the potential function for incompressible nonhardening materials with different creep characteristics in extension and compression. Arguments are presented on the determination of the experimental constants of the material.

1. Let us assume that the creep of nonhardening materials in the uniaxial stress state is described by a relation of the form

$$\eta = B\,|\,\sigma\,|^{n} \tag{1.1}$$

The creep exponent n is assumed to be the same in extension and compression, the creep coefficients B are assumed to be different in extension and compression and hereafter are denoted by B_+ and B_- respectively, η is the creep strain rate, and σ is the stress.

We write the potential function Φ, homogeneous in the stresses, for the creep strain rate in the form

$$\Phi = B_0 (S_2{}^{k} + \beta S_2{}^{k-1.5\lambda} S_3{}^{\lambda})^{(n+1)/2k}, \qquad \eta_{ij} = \partial\Phi / \partial\sigma_{ij} \tag{1.2}$$

Here

$$S_2 = {}^1/{}_2\, \sigma_{ij}{}^{\circ} \sigma_{ij}{}^{\circ}, \qquad S_3 = -\sigma_{ik}{}^{\circ} \sigma_{kj}{}^{\circ} \sigma_{ji}{}^{\circ}, \qquad \sigma_{ij}{}^{\circ} = \sigma_{ij} - {}^1/{}_3 \sigma_{kk} \delta_{ij}$$

Here σ_{ij} are the stress tensor components, η_{ij} are the creep deformation rate deviator components, B_0, k, n, λ are constants of the material.

If we introduce the stress-state angle ξ, where

$$\sin 3\xi = {}^1/{}_2\, \sqrt{3}\, S_3 / S_2{}^{3/2}$$

then (1.2) takes the form

$$\Phi = B_0 S_2{}^{1/2\,(n+1)} [1 + \alpha (\sin 3\xi)^{\lambda}]^{1/2\,(n+1)/k} \tag{1.3}$$

The creep deformation rate deviator components will be

$$\eta_{ij} = \frac{(n+1) B_0 S_2^{1/2\,(n-1)}}{2} [1 + \alpha (\sin 3\xi)^{\lambda}]^{1/2\,(n+1-2k)/k} \left\{ \left[1 + \frac{\alpha (2k - 3\lambda)(\sin 3\xi)^{\lambda}}{2k} \right] \right.$$
$$\left. \times \sigma_{ij}{}^{\circ} - \frac{3}{2} \frac{\lambda\alpha}{k} \left(\frac{3}{S_2} \right)^{1/2} (\sin 3\xi)^{\lambda-1} \left(\sigma_{ik}{}^{\circ} \sigma_{kj}{}^{\circ} - \frac{2}{3} S_2 \delta_{ij} \right) \right\}$$
$$\left(\alpha = \left(\frac{2}{\sqrt{3}} \right)^{\lambda} \beta \right) \tag{1.4}$$

We obtain the following relation for the deviator "similarity phase" ω:

$$\operatorname{tg} \omega = \frac{1}{2S_2} \frac{\partial\Phi / \partial\xi}{\partial\Phi / \partial S_2} = \frac{3}{2} \frac{\lambda\alpha}{k} \frac{(\sin 3\xi)^{\lambda-1} \cos 3\xi}{1 + \alpha (\sin 3\xi)^{\lambda}} \tag{1.5}$$

It is obvious from (1.4) that for materials with different creep characteristics in tension and compression it is necessary that λ be an odd number.

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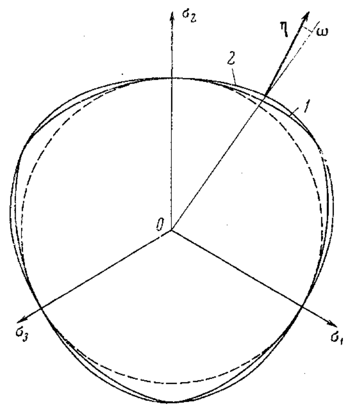

Fig. 1

In many experiments, [4], for example, some axial creep has been observed for pure torsion of the specimens. In order to describe this experimental fact we must on the basis of (1.5) set $\lambda = 1$.

Thus, to specify the potential function (1.3) we must determine the parameters n, α, k, B_0. The creep exponent n is usually found from (1.1) [2]. Further, setting in (1.4) sequentially nonzero $\sigma_{11} = \sigma > 0$ $(\xi = -{}^1/_6\,\pi)$ and $\sigma_{33} = \sigma < 0$ $(\xi = {}^1/_6\,\pi)$ and comparing the resulting relations with (1.1) we obtain

$$\alpha = \frac{A_- - A_+}{A_- + A_+}, \quad A_+ = \left(\frac{3^{(n+1)/2} B_+}{n+1}\right)^{2k/(n+1)}, \quad A_- = \left(\frac{3^{(n+1)/2} B_-}{n+1}\right)^{2k/(n+1)} \qquad (1.6)$$

which together with (1.5) make it possible to find α and k. The remaining constant B_0 is expressed in terms of those already defined

$$B_0 = [{}^1/_2\,(A_- + A_+)]^{(n+1)/2k}$$

We see from these relations that if the creep characteristics in tension and compression are the same $B_+ = B_-$, then $\alpha = 0$ and relations (1.3) and (1.4) become the usual relations [2,5] and then $\omega \equiv 0$.

2. From (1.2) the power W dissipated in the case of creep will be

$$W = \eta_{ij}\sigma_{ij} = (n+1)\Phi \qquad (2.1)$$

In stress space the surface of constant energy dissipation W = const is a cylindrical surface whose axis of symmetry is perpendicular to the deviatoric plane and passes through the coordinate origin. The requirement that the surface W = const be convex imposes certain limitations on the constants α and k, which are found from experiment. In fact, in the ρ, θ, z cylindrical system, where the z axis is aligned with the axis of the cylinder, the angle θ is measured from the direction in the deviatoric plane given relative to the σ_1, σ_2, σ_3 axes by the direction cosines $1/\sqrt{2}$, 0, $-1/\sqrt{2}$ respectively; the trace of the cylinder W = const in the deviatoric plane will be given by the equation

$$\rho = C\,[1 + \alpha\,(\sin 3\theta)^{\lambda}\,]^{-1/2k}, \quad \rho = {}^1/_3\sqrt{3}\,\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2} \qquad (2.2)$$

$$\operatorname{tg}\theta = \frac{1}{\sqrt{3}}\,\frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} = \operatorname{tg}\xi, \quad -\frac{\pi}{6} \leqslant \xi \leqslant \frac{\pi}{6}, \quad z = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sqrt{3}}$$

It follows immediately from (1.6) and (2.2) that $|\alpha| < 1$. For convexity of the surface W = const, we must require that the equation defining the inflection point,

$$\rho^2 + 2\left(\frac{\partial\rho}{\partial\theta}\right)^2 - \rho\,\frac{\partial^2\rho}{\partial^2\theta} = 0,$$

or in expanded form for $\lambda = 1$,

$$(4k^2 - 9)\alpha^2\sin^2 3\theta + 2\,(4k - 9)k\alpha\sin 3\theta + 9\,(1 - 2k)\alpha^2 + 4k^2 = 0 \qquad (2.3)$$

not have a solution. The corresponding analysis of (2.3) is very simple and therefore is not presented here.

The figure shows the sections W = const in the deviatoric plane for the following initial conditions:

$$n = 9, \quad B_+/B_- = 3, \quad k = -3, \quad \alpha = 0.32 \quad \text{(curve 2)},$$
$$n = 9, \quad B_+/B_- = 3, \quad k = 10, \quad \alpha = -0.8 \quad \text{(curve 1)},$$

where the dashed curve shows for comparison the Mises circle. As a consequence of (2.1) the creep strain rate vector will be orthogonal to the surface W = const.

In conclusion we note that a similar analysis with construction of the corresponding potential function can be made for materials whose creep for the uniaxial stress state is described by a relation of the form

$$\eta = K\exp(\varkappa\sigma)$$

with different constants K and $\varkappa$ for tension and compression.

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